Pocket Lamp Brainstorming

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Connection between the pocket lamp and the production dipole - buffer stocks calculation

In the sequel, we shall make a direct use the calculations exposed in the paper by P. Coillard and J.-M. Proth (1983) (COPRO). We assume that we have a generator $\mathcal{G}$ delivering c_1 [A/h] which feeds a light source $\mathcal{S}$ with comsumption c_2 [A/h]. The station $\mathcal{G}$ operates with intermittency and we assume, for simplicity, that it alternates, (i.e. an "ON-OFF" operating mode) with random "ON" time $\tau_{\mathcal{G},ON}$ with average time duration $1/p_1$ and random "OFF" time $\tau_{\mathcal{G},OFF}$ with average time duration $1/r_1$. Similarly for the light source $\mathcal{S}$ we shall assume an intermittent lightening with random "ON" time $\tau_{\mathcal{S},ON}$ with average time duration $1/p_2$ and random "OFF" time $\tau_{\mathcal{S},OFF}$ with average time duration $1/r_2$. For being able to use the COPRO calculations, we make the hypothese that $\tau_{\mathcal{G},ON}$, $\tau_{\mathcal{G},OFF}$, $\tau_{\mathcal{S},ON}$ and $\tau_{\mathcal{S},OFF}$ are independent exponentially distributed r.v.'s with respective parameters p_1, r_1, p_2 and r_2[1].

Now, we may directly use the COPRO results but the situation under study, the interpretation has to be modified. On the long run, we are interested in the portion of time m_{01}^{00} [2] In COPRO m_{01}^{00} indicates that the machine M_1 (here $\mathcal{G}$) is failed, the machine M_2, (here $\mathcal{S}$) is operational and the buffer maximal content h (here the battery maximal capacity) is empty. In this case when the machine M_2 does not deliver production, (here it will imply $\mathcal{S}$ is not enlighten) since it is starved. Hence the operational state is (01) but the effectively observed state is (00), the $\mathcal{S}$ requires electric current but its demand cannot be satisfied since the battery is empty.

All kind of configurations are obviously possible depending on the actual realization one has in mind. As an illustration, let us consider the case where one requires a permanent illumination. In this case $\mathcal{S}$ will be systematically ON and hence $p_2 = 0$. Conversely, we shall assume that $\mathcal{G}$ is alternating between i) random interruptions and ii) in charge states with parameters p_1 and r_1. The relevant situation will generally require that $c_1 >> c_2$ [3]. For this case, we can use directly COPRO subsection 2.3.2.2.2 (page 31). Accordingly, we shall have m_{01}^{00} given by:

[1] This obviously is equivalent to say that coefficients of variations (CV) are equal to unity. Accordingly if in reality, the CV's are less than unity, our calculations will be produce pessimist results, (i.e. the system will actually perform slightly better that calculated and vice-versa.

[2] This notation is adopted to allow a direct match with the COPRO results.

[3] If this is not realized, the battery will stay empty and $\mathcal{S}$ will be ON only when $\mathcal{G}$ will be ON; and even then the lightening might not be optimal since $c_2 > c_1$.

$$\begin{cases} m_{01}^{00}(h) = W(h) D \frac{c_2}{r_1}, \\ \\ D = \frac{p_1}{c_2 - c_1} + \frac{r_1}{c_2}, \end{cases} \tag{1}$$

where D can be found on page 7 in COPRO, with $p_2 = r_2 = 0$ as, in our case, $\mathcal{S}$ is systematically ON. It now remains for us to calculate the normalization factor W(h). For this we have to use the COPRO Eq.(13) together with COPRO Eqs, (40.1) to (40.5). In our situation the normalization will be given by:

$$\begin{cases} F_{01}(h) + F_{11}(h) + M_{11}^{11} + m_{01}^{00} = 1, \\ \\ F_{01}(h) = W(h) \left[e^{DH} - 1 \right], \\ \\ F_{11}(h) = W(h) \frac{c_2}{c_1 - c_2} \left[e^{DH} - 1 \right], \\ \\ M_{11}^{11}(h) = W(h) \frac{c_2}{p_1} D e^{Dh}, \\ \\ m_{01}^{00}(h) = W(h) \frac{c_2 D}{r_1}. \end{cases} \tag{2}$$

Hence using directly Eq.(2), we end with:

$$\begin{aligned} W^{-1}(h) &= \left\{ \left[e^{Dh} - 1 \right] \frac{c_1}{c_1 - c_2} + D \frac{c_2}{p_1} e^{Dh} + D \frac{c_2}{r_1} \right\}, \\ m_{01}^{00}(h) &= \frac{c_2 D}{r_1 \left\{ \left[e^{Dh} - 1 \right] \frac{c_1}{c_1 - c_2} + D \frac{c_2}{p_1} e^{Dh} + D \frac{c_2}{r_1} \right\}}. \end{aligned} \tag{3}$$

At this stage, one has to remember that $m_{01}^{00}(h) \in [0, 1]$ represents the fraction of time that $\mathcal{S}$ is OFF on a long time operation. So Eq.(3) can be used in different manners. For example:

a) **Given the battery capacity h**, one can use Eq.(3) to calculate the fraction of time $\mathcal{S}$ will be OFF, ($\mathcal{S}$ is unlighted).

b) **Given a darkness risk factor $\mathcal{R} \in [0, 1]$**, one can calculate the battery maximal capacity h_s required to hedge this risk. To this aim, one will write:

$$\mathcal{R}(h_s) = m_{01}^{00}(h_s). \tag{4}$$

Interpretation of the results

Absence of battery, (i.e. $h = 0$) and $c_1 > c_2$

When $h = 0$, (i.e. which is equivalent to the situation wher no battery is installed), Eq.(3) reduces to:

$$
\begin{cases}
W^{-1}(0) = Dc_2 \left[\frac{1}{p_1} + \frac{1}{r_1} \right] \\
\\
m_{01}^{00}(0) = \frac{1}{1+\frac{1}{\mathcal{J}_1}} \in [0,1], \qquad \mathcal{J}_1 := \frac{p_1}{r_1}.
\end{cases}
\tag{5}
$$

In view of Eq.(5), we consistently conclude that when $p_1 \to 0 \Rightarrow \mathcal{J}_1^{-1} \to \infty$[4], (i.e. $\mathcal{G}$ virtually never stops), $m_{01}^{00}(0) = 0$ and the light source $\mathcal{S}$ is always ON. Conversely, when $p_1 \to \infty$, the generator $\mathcal{G}$ never operates and therefore, in this limit, we have $\mathcal{J}_1^{-1} = 0 \Rightarrow m_{01}^{00}(0) = 1$ implying that $\mathcal{S}$ is systematically OFF.

Presence of a battery, i.e. $h \neq 0$ **and** $c_1 > c_2$

Now further explore the implications of our expression Eq.(3) for $h \neq 0$. Two regimes have to be differentiated depending on the sign of the parameter D given in Eq.(1). As before, we systematically assume that when $\mathcal{G}$ is ON, it is able to sustain permanent lightening and this even when no battery is present, that is to say that $c_1 > c_2$.

• case $D > 0$. **The generator $\mathcal{S}$ is, on average, able to cover the $\mathcal{S}$ demand**

$$
0 < D = -\underbrace{\frac{p_1}{c_1 - c_2}}_{>0} + \frac{r_1}{c_2} \iff \left(1 + \frac{p_1}{r_1}\right) = (1 + \mathcal{J}_1) < \frac{c_1}{c_2}. \tag{6}
$$

From Eq.(6), we have immediately that $c_2 < \frac{c_1}{1+\mathcal{J}_1}$ and therefore **the average energy generated by $\mathcal{G}$ exceeds the consumption of the lamp $\mathcal{S}$.**

For large battery content h, in view of Eq.(3), we can approximately write:

$$
\begin{cases}
W(h)^{-1} \simeq e^{Dh} \left[\frac{c_1}{c_1 - c_2} + D\frac{c_2}{p_1} \right], \qquad (h >> 1), \\
\\
m_{01}^{00}(h) = W(h)\frac{c_2 D}{r_1} \simeq e^{-Dh} \underbrace{\left\{ \frac{c_2 D}{r_1 \left[\frac{c_1}{c_1 - c_2} + \frac{Dc_2}{p_1} \right]} \right\}}_{:=\Psi(h)} := \psi(h) e^{-Dh}.
\end{cases}
\tag{7}
$$

From Eq.(7), we observe that $m_{01}^{00}(0) \to 0$ for $h \to \infty$. This is intuitively consistent since the average energy generation exceeds the needs and therefore large battery capacities $h >> 1$ will hedge all fluctuations of the energy source.

• case $D < 0$. **The generator $\mathcal{S}$ is, <u>on average</u>, not able to satisafy the $\mathcal{S}$ demand**

$$
0 > D = -\underbrace{\frac{p_1}{c_1 - c_2}}_{>0} + \frac{r_1}{c_2} \iff \left(1 + \frac{p_1}{r_1}\right) = (1 + \mathcal{J}_1) > \frac{c_1}{c_2}. \tag{8}
$$

[4] Remeber that $(p_1)^{-1}$ is the average time the genrator $\mathcal{G}$ is ON .

Eq.(8) implies straightforwardly that $c_2 > \frac{c_1}{1+\mathcal{J}_1}$ and therefore the average energy generated by $\mathcal{G}$ is not sufficient to cover the consumption of the lamp $\mathcal{S}$, (even if we instantaneously have $c_1 > c_2$). In other words, one simply has to adapt our availability, (one has to manage to operate $\mathcal{G}$ more frequently so to reduce p_1) in order to guarantee $D > 0$ and then one will be able to fully benefit fo the presence of the battery.

All potentially remaining configurations not discussed above (i.e. for example those including cases when $p_2 \neq 0$ and $r_2 \neq 0$) can be calculated in a fully similar manner by directly using the COPRO paper.