correlation structure of the predictors. For this reason we assume that the data are being analyzed with standardized predictors and make the following definition:

DEFINITION 8.1 The *variance inflation factor*, VIF_j, associated with the jth predictor is the variance of the estimated coefficient for that predictor divided by the variance if the predictors were orthogonal. That is,

$$VIF_j = \frac{\text{Var}[\hat{\beta}_j]}{\sigma^2} = r^{jj}.$$

Here r^{jj} is the jth diagonal element of the inverse of the correlation matrix of the predictors. Note that VIF is one if the predictors are uncorrelated. ∎

While Definition 8.1 provides a simple means of determining VIF, it does not reveal the source of the problem. For this purpose we examine the diagonal elements of this inverse matrix. For simplicity of notation, we consider $j = 1$ and write the correlation matrix as

$$C = \begin{bmatrix} 1 & r_1^T \\ r_1 & C_1 \end{bmatrix}, \tag{8.11}$$

where r_1^T denote the row-vector of correlations of the first predictor with the remaining ones, and C_1 denotes the correlation matrix for the remaining predictors. Note that C is the coefficient matrix for the normal equations in standard form hence the diagonal elements of the inverse matrix, when multiplied by σ^2, give the variance of the estimates. Using the results on the inverse of a partitioned matrix, we see that

$$r^{11} = \frac{1}{(1 - r_1^T C_1^{-1} r_1)} = \frac{1}{(1 - R_1^2)} . \tag{8.12}$$

Note that $C_1^{-1} r_1$ is the vector of coefficient estimates for the regression of the first predictor on the remaining $m - 1$ predictors, and hence R_1^2 is the coefficient of determination for that regression. In general then, we see that the jth variance inflation factor is given by

$$VIF_j = \frac{1}{(1 - R_j^2)}, \tag{8.13}$$