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# Mathematical formula for spatial LCA computation

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This is the mathematical supplement for the working manuscript “ [Systematic computational framework for spatial life cycle assessment](#)”

## 1. Site-Generic LCA

$$H = CBA^{-1}f$$

where,

C is generic impact assessment characterization factors (Geography: generic/global or continental level; e.g. CML, IPCC);

B is environmental intervention matrix (Geography: usually generic/global or country level)

A is technology matrix (Geography: usually generic/global or country level)

f is demand matrix or functional unit

## 2. Methods for spatial LCA approach

2.1. *Focusing on (usually partially) regionalizing environmental flow matrix alone*

$$H = CB_R A^{-1}f$$

e.g. Quantis water database ; ecoinvent version 3, Water Footprint Network

2.2. *Focusing on (usually partially) regionalizing technology matrix alone*

$$H = CBA_R^{-1}f$$

e.g. Quantis water database partially regionalized; ecoinvent version 3

2.3. *Focusing on both technology and environmental flow matrix*

$$H = CB_R A_R^{-1}f$$

e.g. USDA Agricultural database; ecoinvent version 3

2.4. *Focusing on impact assessment*

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$$H = C_R B A^{-1} f$$

e.g. IMPACT World+, Water Scarcity

### 2.5. *Focusing on both impact assessment and environmental flows*

$$H = C_R B_R A^{-1} f$$

where,  $C_R$  is spatial impact assessment characterization factors (e.g. IMPACT World+, Stephan Pfister water use); it normally can be classified into the following categories ( Jose Potting, et al)

- Site-dependent
- Archetype
- Site-Specific

$C_R$  is spatial biosphere inventory data (e.g. irrigation water on state/watershed/ GIS data set (0.5 x 0.5 degree grid) level)

- Site-dependent
- Archetype
- Site-Specific

### 2.6. *Focusing on harmonizing spatial scale incompatibility between impact assessment and environmental flows*

$$H = C_R X_R B_R A^{-1} f$$

$X_R$  is conversion approach for matching difference between regionalized impact assessment and regionalized inventory flows.

[Chris Mutel & Stefanie Hellweg \(2014\)](#) proposed the following classification:

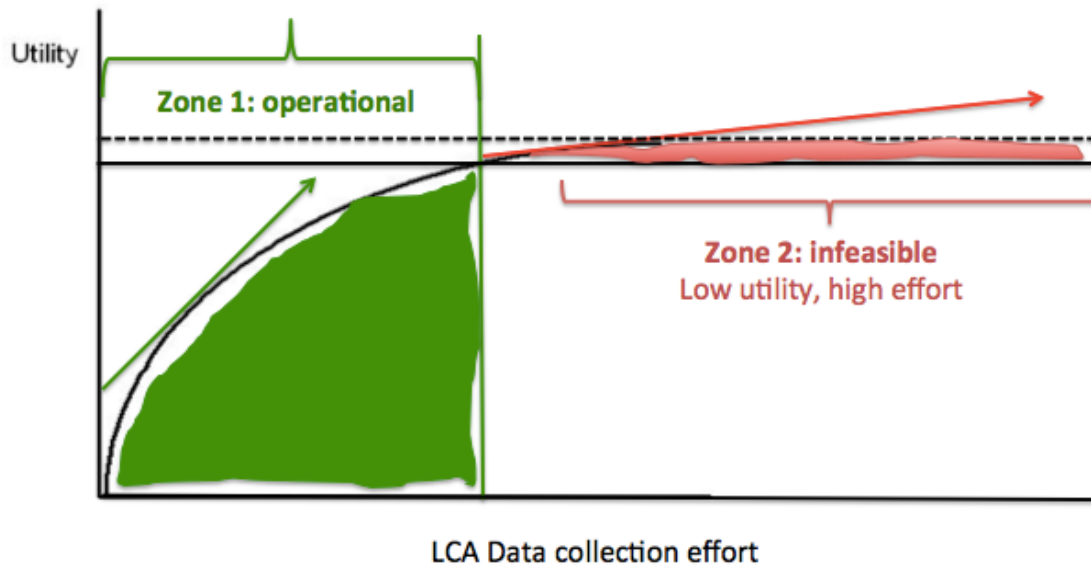
- Shared spatial scale
- Allocation for spatial scales incompatibility
  - Area allocation
  - Background loading approach
  - Inventory extension

## 3. **Proposed step-wise regionalization approach for LCA**

### 3.1. *Law of diminishing marginal utility*

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Regionalized LCA is often criticized for its impracticality and huge data collection efforts. In reality, the most important question is to identify operational zone for regionalization that balances data quality (or utility) and efforts/ cost for data collection as shown in the figure below.



### 3.2. Spatial variability in LCA data for providing similar products or services

#### 3.2.1. Variability of LCIA characterization factors due to location difference

Quite often location information alone could cause 3 order of magnitude of difference (e.g for eutrophication impact or 2 order of magnitude of difference for water scarcity impact assessment).

#### 3.2.2. Variability of technological and environmental properties due to location difference (or management practice for agriculture)

In contrast, spatial variability of inventory activity data due to technological and environmental property difference are often considerably lower (especially for industrial products, often less than < 100% ), even for bio-products, e.g agricultural inputs and emissions, it usually vary less than 1 order of magnitude.

*Therefore, there is a necessity to separate geo-information itself (data collection effort is low) and activity input-output inventory data set (data collection effort is high and often unavailable) into two matrixes when performing regionalized LCA.*

### 3.3. Use location information for impact assessment regionalization

$$H = C_R X_R B_R (A_G)^{-1} f$$

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where  $G$  is geo-information probability distribution matrix for activities.

#Most relevant to tackle the 2 or 3 order of magnitude of variability caused by impact assessment; low to medium effort #

#### 3.4. Refine spatial scale of inventory data

$$H = C_R X_R B_R (A_R G)^{-1} f$$

Expanding input-output inventory data set. Especially for energy and agricultural/bio-products, spatial variability of techno-sphere often cannot be ignored. Sometimes, it needs splitting one activity from aggregated country level to state or even smaller level.

# Most relevant to tackle the 1 order of magnitude of variability caused by spatial variation of inventory data; medium to high effort). #

#### 3.5. Techniques for identification of priorities for regionalization

To identify most material activities/ spatial scales to regionalize through sensitive test based on spatial impact assessment variability of impact of environmental flows of interests as well as regional difference of technological and environmental properties of production systems of interests.

(To be continued)

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## Appendix: Comparison of methods for spatial LCA

Focus of regionalization	Bio-	Techno-	Bio- & Techno-	LCIA	Bio- & LCIA	Scale harmonization	Integrated-location	Integrated-Location & Properties
Reference Section in this paper	2.1	2.2	2.3	2.4	2.5	2.6	3.2.1	3.2.2
Available methodology or example	Quantis water database; WFN	Quantis water database;	USDA LCA Digital common; ecoinvent 3	Stephan Pfister; IMPACT World+; Et al.	Water database + Pfister; WFN	Chris Mutel & Stefanie Hellweg	This paper	This paper
Data requirements	Environmental flows per process	Commodity per process	Commodity & Environmental flows	Location information	Environmental flows per process+ Location	Spatial scale of process and impact assessment	Location information for selected process	Location + commodity & environmental flows for selected process
Time and Labour intensity	Medium to high	Medium to High	Very high	N.A	Medium to high	N.A	Low	Low to medium
Uncertainty to regionalization	Depends (low – high)	Depends (low – high)	Low	N.A	Depends (low – high)	N.A	Low	Low
Systematic/completeness	Low	Low	High	N.A	Low	N.A	Medium	High
Reliability	Low	Low	High	N.A	Low	N.A	Medium to High	High
Regionalization completeness	Medium to poor	Medium to poor	Medium to High	N.A	Low to medium	N.A	Medium to High	Medium to High
Simplicity of application	Simple	Simple	Simple	Simple to medium	Simple to medium	Complex	Simple	Medium to complex
Available software tools	Conventional: SimaPro, Gabi, Quantis SUITE et al.	Conventional: SimaPro, Gabi, Quantis SUITE et al.	Conventional: SimaPro, Gabi, Quantis SUITE Et al.	Excel; Quantis SUITE	Excel; Quantis SUITE	GIS- Aided	Depends: Conventional (+GIS)	TBD; Matlab/R+GIS;